●●●●○ Level 4 · Mathematics M4.2

Percentages, proportion and growth

Use percentages to find changes, and see how amounts grow or shrink over time.

Increase and decrease an amount with a multiplier, find a percentage change, and work back to the original amount. Compare simple and compound interest, model growth and decay, combine ratios, and write and graph direct and inverse proportion.

YOU ARE READY FOR THIS BOOK IF

You can find 15% of 80 without a calculator.

This book	**M4.2 Percentages, proportion and growth**
One step down	M3.4 Ratio and proportion

Contents

M4.2.1 Percentage increase and decrease

M4.2.2 Finding the original amount

M4.2.3 Simple and compound interest

M4.2.4 Growth and decay

M4.2.5 Harder ratio problems

M4.2.6 Proportion as a formula

M4.2.7 Graphs of proportion

Key words, A to Z

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M4.2.1 CHAPTER 1

Percentage increase and decrease

How do I add 12% to 250 in one step?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Add 12 to 250 to get 262.
- ☐ **B** Multiply 250 by 1.12 to get 280.
- ☐ **C** Multiply 250 by 0.12 to get 30.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Fractions, decimals and percentages, M3.3.8. If finding a percentage of an amount looks new, read this first.
- Level 3, Fractions, decimals and percentages, M3.3.6. If 250 × 1.12 looks new, read this first.

BY THE END YOU CAN

- Increase 250 by 12% using the multiplier 1.12
- Find the percentage change from 80 to 92

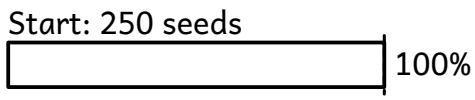
A packet holds 250 seeds. A new packet holds 12% more. How many seeds is that?

Percent means out of 100. So 12% is $^{12}/_{100}$, which is 0.12. 12% of 250 is 0.12 × 250 = 30. Add that to the start: 250 + 30 = 280.

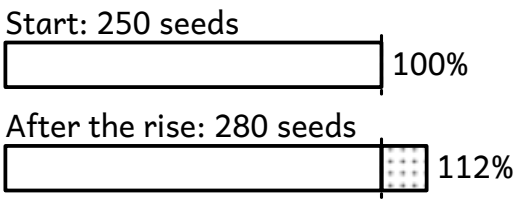
That took two steps. There is a way to do it in one.

DRAW IT FIRST

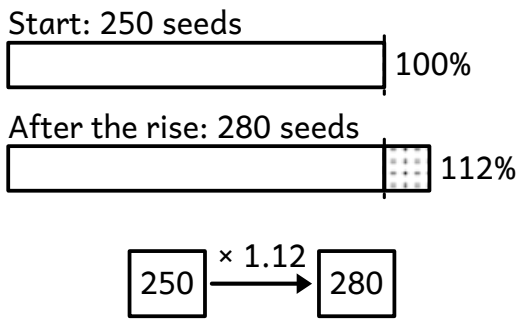
Add the rise to the start



1 The start is 100% of the seeds. That bar is 250 seeds.



2 The rise adds 12% more. The new bar is 112% of the start, and the extra piece is dotted.



3 112% is 1.12 as a decimal. One multiplication takes 250 to 280.

The start is 100%. A rise of 12% makes 100% + 12% = 112%. As a decimal, 112% is 1.12. The number 1.12 is the **multiplier**. Multiply by it, and the rise is done.

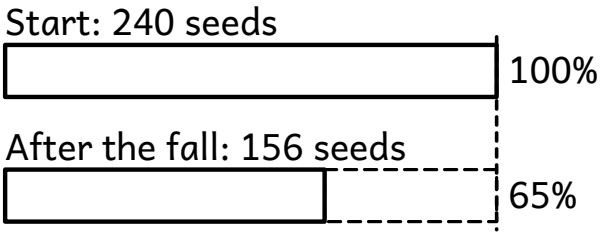
WORKED EXAMPLE

Increase 250 by 12% using a multiplier.

1. A rise of 12% makes 100% + 12% = 112% of the start.
2. 112% as a decimal is 1.12. This is the multiplier.
3. Multiply the start by the multiplier: 250 × 1.12 = 280.

Answer: 250 × 1.12 = 280

A fall works the same way. A fall of 35% leaves 100% − 35% = 65% of the start. As a decimal that is 0.65. The multiplier for a fall is below 1.



A fall of 35% leaves 65% of the start. The missing 35% is the dashed part.

WORKED EXAMPLE

Decrease 240 by 35%.

1. A fall of 35% leaves 100% − 35% = 65% of the start.
2. 65% as a decimal is 0.65. This is the multiplier.
3. Multiply: 240 × 0.65 = 156.

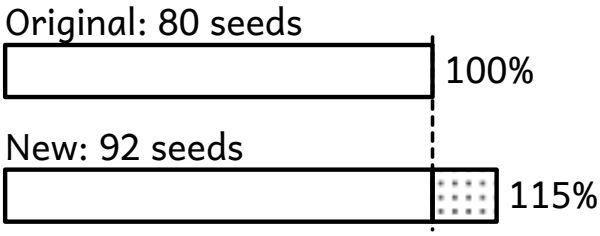
Answer: 240 × 0.65 = 156

Change	Multiplier
Rise of 5%	1.05
Rise of 12%	1.12
Fall of 15%	0.85
Fall of 40%	0.6

A rise adds to 1. A fall takes away from 1.

Sometimes you are given the old and the new amount and asked how much it changed. A **percentage change** says the change as a percent of the amount you started with.

Work out the change: new minus original. Divide it by the original. Multiply by 100. As a formula: percentage change = (new − original) ÷ original × 100.



The dotted piece is the change, 12 seeds. It is measured against the original, which is 100%.

WORKED EXAMPLE

Find the percentage change from 80 to 92.

1. The change is new minus original: 92 − 80 = 12.
2. Divide the change by the original: 12 ÷ 80 = 0.15.
3. Multiply by 100: 0.15 × 100 = 15.
4. The amount rose by 15%.

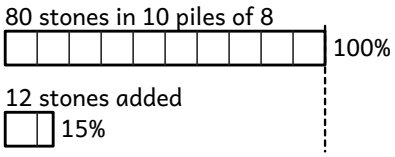
Answer: A rise of 15%

SEE IT THREE WAYS

With things

Take 80 stones. Put them in 10 piles of 8. Add 12 more stones. That is one more pile and half a pile.

As a picture



In symbols

12 ÷ 80 × 100 = 15%.

TRY IT

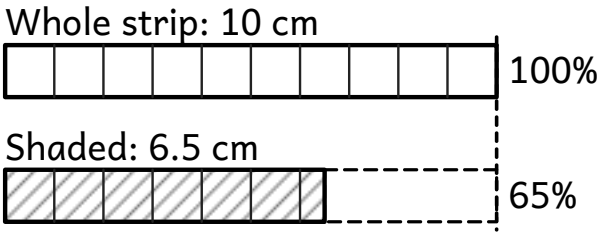
Cut a strip

You need: paper, a ruler, a pencil.

1. Draw a strip 10 cm long. Let the whole strip stand for 100%. Mark every 1 cm.
2. Shade the first 6.5 cm from the left. Leave the last 3.5 cm white.

What you will see: The shaded part is 65% of the strip. A fall of 35% leaves 65%, so the multiplier is 0.65.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~To increase 250 by 12%, add 12: 250 + 12 = 262.~~

Why it feels right: The words say "12" and "add", so it feels natural to add the 12 you can see.

The test that breaks it: Work out 12% of 250. It is 0.12 × 250 = 30, not 12. The rise is 30 seeds, so 262 is too low.

What is true: The percent is a fraction of the start, not a number to add. Use the multiplier: 250 × 1.12 = 280.

A WRONG IDEA MANY PEOPLE HAVE

~~The amount falls from 125 to 100, so the fall is 25 out of 100, which is 25%.~~

Why it feels right: The new amount is the one you are looking at, and 100 makes the sum look like a percent already.

The test that breaks it: A fall of 25% would leave 125 × 0.75 = 93.75, not 100. Check by multiplying.

What is true: Divide by the original amount, 125. The change 25 ÷ 125 = 0.2, so the fall is 20%.

KEY WORDS

percent
out of 100. So 12% is 12 out of 100, or 0.12.

multiplier
the number you multiply by to change an amount, such as 1.12 for a rise of 12%.

percentage change
the change divided by the original amount, times 100.

BACK TO YOUR GUESS

The answer is B: Multiply 250 by 1.12 to get 280.

12% of 250 is 30, and that is what 0.12 gives. The rise adds 30 to the 250, so the new amount is 280. Multiplying by 1.12 does both steps at once.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which multiplier increases an amount by 8%?

   A 0.08

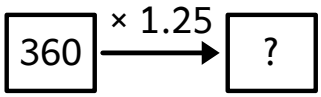
   B 1.8

   C 1.08

   D 0.92

   Answer at the back of the book.

2. The picture shows 360 being increased by 25%. What number goes in the empty box?



   A 450

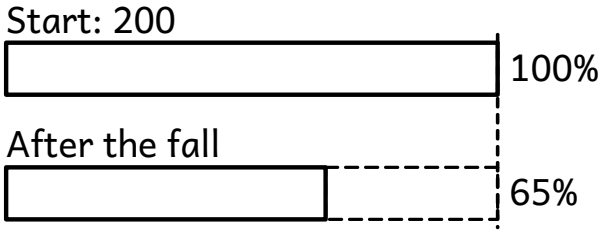
   B 385

   C 90

   D 270

   Answer at the back of the book.

3. Decrease 200 by 35%. The picture shows the bars.



A 165

B 70

C 270

D 130

Answer at the back of the book.

4. What is the percentage change from 50 to 56?

A 6%

B 12%

C 10.7%

D 112%

Answer at the back of the book.

5. An amount goes from 160 down to 120. What is the percentage change?

A A fall of 33.3%

B A fall of 75%

C A rise of 25%

D A fall of 25%

Answer at the back of the book.

6. Increase 160 by 15%.

Answer at the back of the book.

7. A plant grows from 40 cm to 46 cm. What is the percentage increase? Type the number only.

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Rise: multiply by 1 + the rate. Fall: multiply by 1 − the rate. Percentage change = change ÷ original × 100. Always divide by the original.

Draw: A bar of 100% and a longer bar of 112%, with an arrow labelled × 1.12 between them.

M4.2.2 CHAPTER 2

Finding the original amount

> **A herd grew by 25% and now has 150 goats. How many goats did it have before?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Take 25% off 150, which leaves 112.5.
- ☐ **B** Take 25 off 150 to get 125.
- ☐ **C** Divide 150 by 1.25 to get 120.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Fractions, decimals and percentages, M3.3.8. If finding a percentage of an amount looks new, read this first.
- Level 3, Fractions, decimals and percentages, M3.3.6. If 80 × 1.2 looks new, read this first.

BY THE END YOU CAN

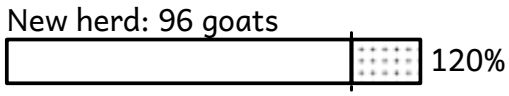
- Find the original amount after a 20% rise gives 96
- Explain why a 10% rise then a 10% fall is a loss

Last year a herd had some goats. It grew by 20% and now has 96 goats. We want the number it started with, the **original amount**.

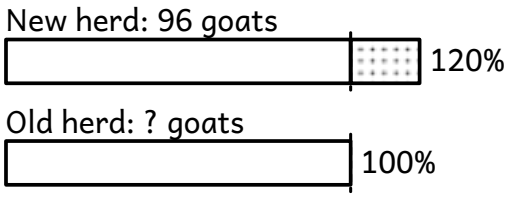
The start is 100%. After a rise of 20%, the herd is 120% of the start. So 96 goats is 120% of the original amount.

DRAW IT FIRST

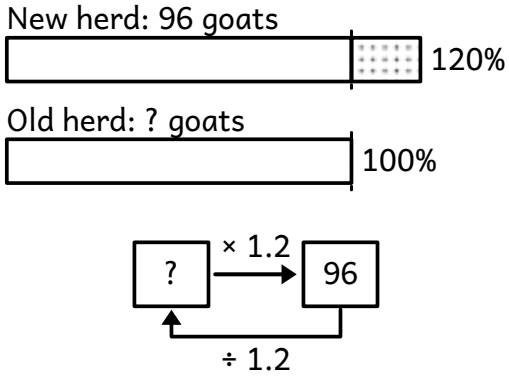
Work back from the new amount



1 The new herd is 96 goats. That is 120% of the original.



2 The old herd is the shorter bar. We do not know it yet.



3 The rise multiplied the old herd by 1.2. To go back, divide the new herd by 1.2.

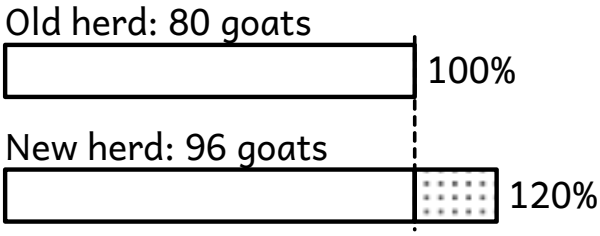
A rise multiplies by 1.2. Going back undoes the multiplying, so divide by 1.2. Divide by the multiplier, whether the change was a rise or a fall. To divide by 1.2, multiply both numbers by 10 first: 96 ÷ 1.2 = 960 ÷ 12 = 80.

WORKED EXAMPLE

A herd grew by 20% and now has 96 goats. Find the original amount.

1. The new amount is 100% + 20% = 120% of the original.
2. 120% as a decimal is 1.2. So original × 1.2 = 96.
3. Go back by dividing by the multiplier: 96 ÷ 1.2 = 80.
4. Check: 80 × 1.2 = 96. So the herd had 80 goats before.

Answer: 96 ÷ 1.2 = 80



The rise is 20% of the old herd: 16 goats. It is not 20% of the new herd.

A WRONG IDEA MANY PEOPLE HAVE

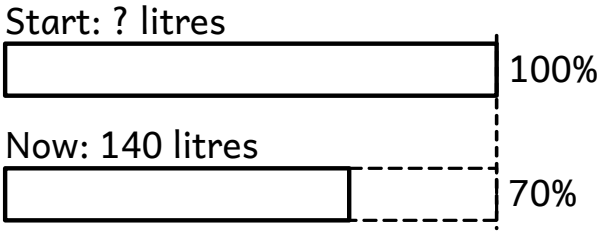
~~**To undo a rise of 20%, take 20% off 96.**~~

Why it feels right: A rise and a fall of the same percent sound like opposites that cancel.

The test that breaks it: 20% of 96 is 19.2, and 96 − 19.2 = 76.8. Now check by rising 20%: 76.8 × 1.2 = 92.16, not 96.

What is true: The rise was 20% of the old amount, 80, and not 20% of 96. Divide by 1.2 to reach 80.

A fall works the same way. The multiplier is below 1, and you still divide by it.



140 litres is 70% of the start.

WORKED EXAMPLE

A tank lost 30% of its water and now holds 140 litres. How much did it hold at the start?

1. A fall of 30% leaves 100% − 30% = 70% of the start.
2. 70% as a decimal is 0.7. So start × 0.7 = 140.
3. Divide by the multiplier: 140 ÷ 0.7 = 200.
4. The tank held 200 litres. Check: 200 × 0.7 = 140.

Answer: 140 ÷ 0.7 = 200

Now a puzzle. A herd of 200 goats grows by 10%. Then it falls by 10%. Is it back where it began?



The herd ends at 198, which is 2 below where it began.

It is not back. The rise of 10% was worked out on 200. The fall of 10% was worked out on 220, which is a bigger amount. A bigger amount loses more.

WORKED EXAMPLE

A herd of 200 goats grows by 10%, then falls by 10%. How does the end compare with the start?

1. A rise of 10% has multiplier 1.1: 200 × 1.1 = 220.
2. A fall of 10% has multiplier 0.9: 220 × 0.9 = 198.
3. One multiplier does both: 1.1 × 0.9 = 0.99.
4. The herd ends at 0.99 of the start, which is 1% less.

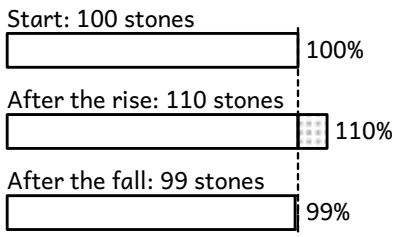
Answer: Less, by 1%

SEE IT THREE WAYS

With things

Take 100 stones. Add 10%: that is 10 more, so 110 stones. Now take away 10% of the 110: that is 11 stones. 99 are left.

As a picture



In symbols

100 × 1.1 × 0.9 = 99.

TRY IT

Try any number

You need: paper, a pencil.

1. Write any whole number you like. Here we use 300.
2. Raise it by 10%. Write the answer.
3. Lower that answer by 10%. Write the new answer.

What you will see: You end below where you began. With 300 you reach 297. Try other numbers: the end is always 0.99 of the start.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~A rise of 10% and then a fall of 10% cancel, so you end where you began.~~

Why it feels right: Plus 10 and minus 10 do cancel. The same number appears twice, so it feels the same here.

The test that breaks it: Take 100. A rise of 10% gives 110. A fall of 10% takes off 10% of 110, which is 11. That leaves 99, not 100.

What is true: Each percent is of a different amount. Multiply the multipliers: 1.1 × 0.9 = 0.99.

KEY WORDS

original amount
the amount before the change. It is 100%.

BACK TO YOUR GUESS

The answer is C: Divide 150 by 1.25 to get 120.

The 25% rise was worked out on the old herd, not on 150. The new herd is 125% of the old herd, so divide 150 by 1.25. That gives 120. Check: 120 × 1.25 = 150.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. After a 40% rise an amount is 210. What was the original amount?

   A 126

   B 170

   C 294

   D 150

   Answer at the back of the book.

2. After a 20% fall an amount is 64. What was it before?

   A 76.8

   B 80

   C 84

   D 51.2

   Answer at the back of the book.

3. After a 25% fall an amount is 90. What was it before?

   A 112.5

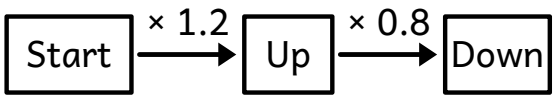
   B 115

   C 120

   D 67.5

   Answer at the back of the book.

4. A rise of 20% is followed by a fall of 20%. Which single multiplier does the same job?



A 0.96

B 1

C 0.8

Answer at the back of the book.

5. An amount rises by 20% and then falls by 20%. Compared with the start, the end amount is:

A the same as the start

B 4% more than the start

C 20% less than the start

D 4% less than the start

Answer at the back of the book.

6. **From an earlier chapter**

Find the percentage change from 60 to 75. Type the number only.

Answer at the back of the book.

7. After a 15% rise an amount is 345. Find the original amount.

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

To go back from a new amount, divide by the multiplier. Never take the same percent off the new amount. A rise then a fall of 10% gives 0.99.

Draw: Two boxes joined by an arrow labelled × 1.2 above and a return arrow labelled ÷ 1.2 below.

M4.2.3 CHAPTER 3

Simple and compound interest

> **Two savings clubs both add 5% a year to the coins I put in. Why does one of them give more?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** One adds 5% of the start amount each year. The other adds 5% of the amount that has grown.
- ☐ **B** They cannot differ. 5% a year is 5% a year.
- ☐ **C** The one that ends with more must have a higher rate.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Fractions, decimals and percentages, M3.3.8. If finding a percentage of an amount looks new, read this first.
- Level 3, Negative numbers, factors and powers, M3.1.7. If a power such as 1.05 × 1.05 × 1.05 looks new, read this first.

BY THE END YOU CAN

- Work out simple interest on 2 000 coins at 4% a year
- Find the value after 3 years of 5% compound interest
- Compare simple and compound interest in a table over 10 years

A saver puts 2 000 coins into a savings club. Each year the club adds **interest**, a percent of the savings. Here the rate is 4% a year.

Simple interest is the same amount every year. It is always 4% of the start amount. 4% of 2 000 is 80, so the saver gains 80 coins each year.

Year	Interest	Total
1	80	2 080
2	80	2 160
3	80	2 240
4	80	2 320
5	80	2 400

Simple interest on 2 000 coins at 4% a year. The interest is the same every year.

WORKED EXAMPLE

Work out the simple interest on 2 000 coins at 4% a year for 5 years.

1. One year of interest: 4% of 2 000 = 0.04 × 2 000 = 80.
2. 5 years of interest: 5 × 80 = 400.
3. The total is 2 000 + 400 = 2 400 coins.

Answer: 400 coins of interest

To find simple interest, multiply the start amount by the rate as a decimal, and then by the number of years.

Compound interest is different. Each year the interest is worked out on the whole amount, including interest already added. The amount is multiplied by the same multiplier every year. For 5% the multiplier is 1.05.

Year	Start	Added	End
1	8 000	400	8 400
2	8 400	420	8 820
3	8 820	441	9 261

8 000 coins at 5% compound interest. The interest grows because the start of each year is bigger.

WORKED EXAMPLE

8 000 coins earn 5% compound interest a year. How many coins are there after 3 years?

1. Each year the amount is multiplied by 1 + 0.05 = 1.05.
2. After year 1: 8 000 × 1.05 = 8 400.
3. After year 2: 8 400 × 1.05 = 8 820.
4. After year 3: 8 820 × 1.05 = 9 261.

Answer: 9 261 coins

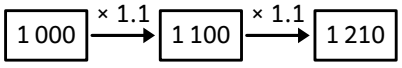
The three steps multiply by 1.05 three times. That is 1.05 × 1.05 × 1.05, written 1.05^3. So the value after n years is the start amount times the multiplier, n times over.

SEE IT THREE WAYS

With things

Put in 1 000 coins at 10% a year. After one year add a tenth: 1 100. After year two add a tenth of 1 100: 1 210.

As a picture



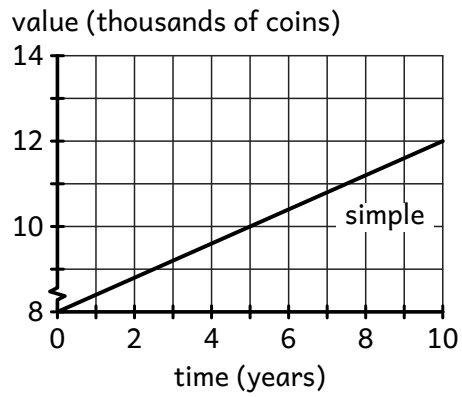
In symbols

1 000 × 1.1 × 1.1 =
1 000 × 1.21 =
1 210.

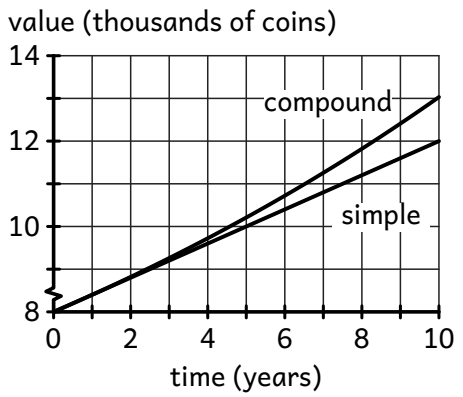
Now compare the two over 10 years. Take 8 000 coins at 5% a year, both ways. In year 1 they are equal. After that, compound interest pulls ahead.

DRAW IT FIRST

Simple and compound on one graph



1 Simple interest adds the same amount each year, so its graph is a straight line.



2 Compound interest adds more each year, so its graph bends upward. The curve starts on the line, then rises above it.

Year	Simple	Compound
0	8 000	8 000
1	8 400	8 400
2	8 800	8 820
3	9 200	9 261
4	9 600	9 724
5	10 000	10 210
6	10 400	10 721
7	10 800	11 257
8	11 200	11 820
9	11 600	12 411
10	12 000	13 031

Compound values are rounded to the nearest coin. The two match in year 1, and then the gap widens each year.

WORKED EXAMPLE

8 000 coins earn 5% a year for 10 years. How many more coins does compound interest give than simple interest?

1. Simple interest for 10 years: 8 000 × 0.05 × 10 = 4 000.
2. So the simple total is 8 000 + 4 000 = 12 000.
3. A calculator multiplies 8 000 by 1.05 ten times and gives 13 031, to the nearest coin.
4. Compound gives more by 13 031 − 12 000 = 1 031.

Answer: 1 031 coins more

TRY IT

Make your own table

You need: paper, a pencil.

1. Start with 1 000 coins and a rate of 10% a year. Draw three columns: year, simple and compound.
2. For simple interest, add 100 each year.
3. For compound interest, multiply the last amount by 1.1 each year. Round to the nearest coin.

What you will see: The two columns match in year 1. In year 2 compound is already ahead, with 1 210 against 1 200, and the gap grows.

If you cannot try it, this is what happens.

Year	Simple	Compound
0	1 000	1 000
1	1 100	1 100
2	1 200	1 210
3	1 300	1 331
4	1 400	1 464

A WRONG IDEA MANY PEOPLE HAVE

~~8 000 coins at 5% compound interest for 3 years earns 3 × 5% = 15%, so the total is 9 200.~~

Why it feels right: Three years of 5% looks like 15%, and it is the answer simple interest would give.

The test that breaks it: Work year by year: 8 000 → 8 400 → 8 820 → 9 261. The total is 9 261, which is more than 9 200.

What is true: Each year's interest is worked out on a bigger amount. Multiply by 1.05 three times, not by 1.15 once.

KEY WORDS

interest
extra amount added to savings each year, worked out as a percent.

simple interest
interest that is always a percent of the start amount, so it is the same every year.

compound interest
interest worked out on the amount so far, including interest already added.

BACK TO YOUR GUESS

The answer is A: One adds 5% of the start amount each year. The other adds 5% of the amount that has grown.

The rate can be the same and the totals still differ. Simple interest is always worked out on the start amount. Compound interest is worked out on the amount that has grown, so it earns interest on interest.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Work out the simple interest on 2 500 coins at 6% a year for 4 years.

   A 150

   B 656

   C 600

   D 3 100

   Answer at the back of the book.

2. 4 000 coins earn 5% compound interest a year. How many coins are there after 2 years?

   A 4 410

   B 4 400

   C 4 200

   Answer at the back of the book.

3. Which multiplier is used each year for 4% compound interest?

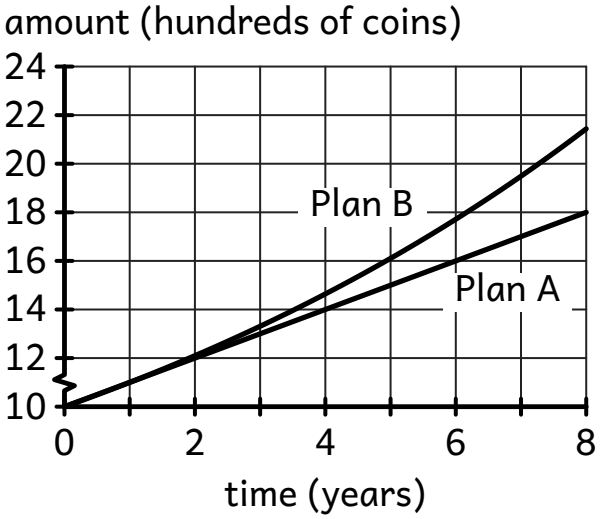
   A 0.04

   B 1.04

   C 1.4

   Answer at the back of the book.

4. The graph shows two plans that start with the same amount and the same rate. One adds simple interest and one adds compound interest. Which plan adds compound interest?



A Plan A

B Both of them

C You cannot tell from a graph

D Plan B

Answer at the back of the book.

5. Two savings clubs have the same start amount and the same rate. One adds simple interest and one adds compound interest. After one year, which club has more?

A The compound club

B The simple club

C They have the same

Answer at the back of the book.

6. **From an earlier chapter**

After a 20% fall an amount is 480 coins. What was it before? Type the number only.

Answer at the back of the book.

7. Work out the simple interest on 3 000 coins at 3% a year for 4 years.

   Answer at the back of the book.

8. A savings club adds 10% compound interest a year. How many coins are there after 2 years if 2 000 coins are put in?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Simple interest: start × rate × years, the same each year.
Compound interest: multiply by 1 + rate once for every year.
Compound is ahead after year 1.

Draw: A graph with a straight line named simple and a curve named compound above it.

Growth and decay

The fish in a lake fall by 15% every year. Will the lake ever have no fish at all?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes, after about 7 years, because 15% a year for 7 years is about 100%.
- ☐ **B** No. Each year it keeps 85% of the fish, so some always remain.
- ☐ **C** Yes, after exactly 15 years.

IF THIS FEELS HARD, READ THIS FIRST

- Level 4, Percentages, proportion and growth, M4.2.3. If multiplying by the same number every year looks new, read this first.
- Level 3, Negative numbers, factors and powers, M3.1.7. If a power such as 0.85^3 looks new, read this first.

BY THE END YOU CAN

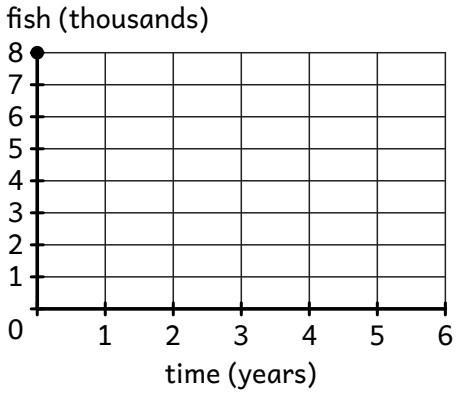
- Use the multiplier 0.85 for a 15% fall each year
- Model a goat herd that grows by 10% a year

Ana studies a lake. It has 8 000 fish. Each year the number falls by 15%. How many fish are left after 3 years?

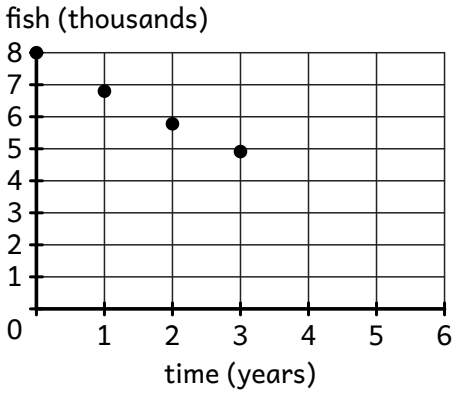
A fall of 15% leaves 85% of the fish. As a decimal that is 0.85. Each year the lake keeps 0.85 of what it had. So each year you multiply by 0.85 again.

DRAW IT FIRST

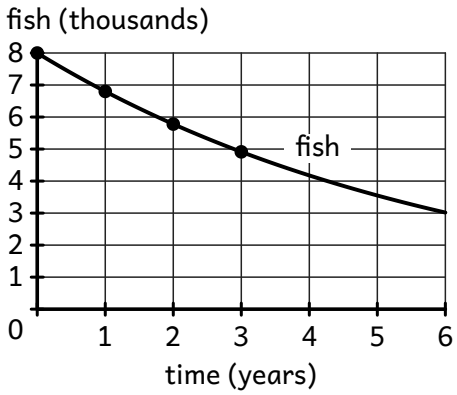
Watch the fish fall



1 At the start the lake has 8 000 fish, which is 8 on this scale of thousands.



2 After 1, 2 and 3 years the dots are lower each time. The drops get smaller, because 15% of a smaller number is smaller.



3 The curve joins the dots. It falls fast at first, then more slowly, and it stays above zero.

A quantity that falls by the same percent every year is in **decay**. A quantity that rises by the same percent every year is in **growth**. Both are described by a **model**: a rule that says how the amount changes.

WORKED EXAMPLE

A lake has 8 000 fish. The number falls by 15% each year. How many fish are there after 3 years?

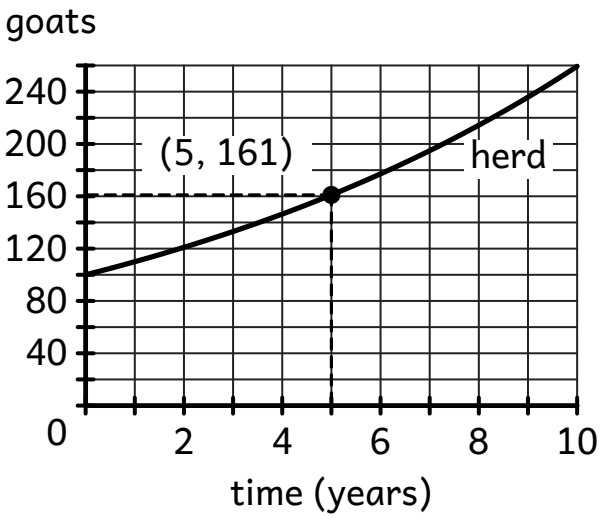
1. A fall of 15% leaves 85%, so the yearly multiplier is 0.85.
2. After year 1: 8 000 × 0.85 = 6 800.
3. After year 2: 6 800 × 0.85 = 5 780.
4. After year 3: 5 780 × 0.85 = 4 913.

Answer: 4 913 fish

Year	Fish
0	8 000
1	6 800
2	5 780
3	4 913
4	4 176
5	3 550
6	3 017

Each row is 8 000 × 0.85 once for every year, rounded to the nearest whole fish. The fall is 1 200 in year 1 but only 533 in year 6.

Growth works the same way, with a multiplier above 1. A herd of 100 goats grows by 10% each year. The yearly multiplier is 1 + 0.1 = 1.1.



Growth bends upward. Each year the herd gains more than the year before.

WORKED EXAMPLE

A herd of 100 goats grows by 10% each year. How many goats are there after 5 years, to the nearest goat?

1. A rise of 10% gives a yearly multiplier of 1.1.
2. Five years means multiplying by 1.1 five times: $1.1^5 = 1.61051$.
3. The herd is $100 \times 1.61051 = 161.051$.
4. Goats are whole animals, so round to the nearest goat: 161.

Answer: 161 goats

The model gives parts of a goat. We compare the model's numbers with 150 and only round at the end.

WORKED EXAMPLE

The same herd starts at 100 goats. In which year does it first pass 150 goats?

1. Year 3 is below 150: $100 \times 1.1^3 = 133.1$.
2. Year 4 is below 150: $133.1 \times 1.1 = 146.41$.
3. Year 5 is above 150: $146.41 \times 1.1 = 161.051$.
4. So the herd first passes 150 goats in year 5. In year 5 the model gives 161.051, which is about 161 goats.

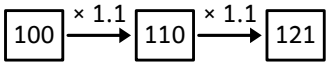
Answer: Year 5

SEE IT THREE WAYS

With things

A herd of 100 goats. After one year add a tenth: 110. After year two add a tenth of that: 121.

As a picture



In symbols

$100 \times 1.1 \times 1.1 = 121$.

TRY IT

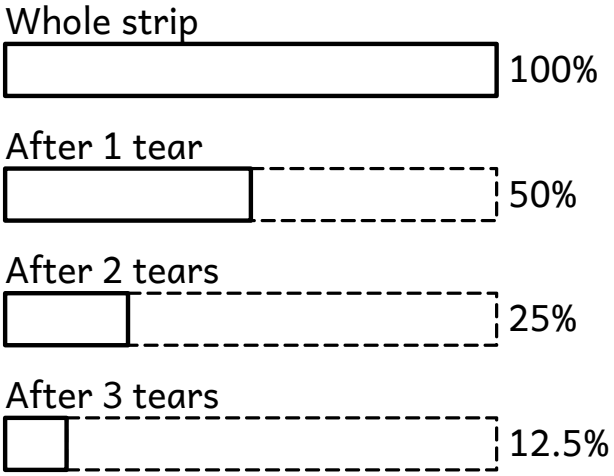
Tear a strip of paper in half

You need: a strip of paper.

1. Let the whole strip stand for 100%. Tear it in half and keep one piece.
2. Tear the piece in half again and keep one piece. Do this three times in all.

What you will see: The pieces are 50%, 25% and 12.5% of the strip. Each tear multiplies by 0.5, a fall of 50%. In the model you can tear for ever and never reach nothing. A real strip becomes too small to tear.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~A fall of 15% a year takes about 7 years to wipe out the fish, because 7 × 15% is about 100%.~~

Why it feels right: Seven falls of 15 take off about 100. That is right for 15 fish a year, but not for 15 percent.

The test that breaks it: A fall of 15% takes 15% of what is left, which gets smaller every year. After 7 years the lake still has about 2 565 of its 8 000 fish, which is 32%.

What is true: Multiply by 0.85 once for every year: 0.85 × 0.85 and so on. The number keeps falling but never reaches zero.

A WRONG IDEA MANY PEOPLE HAVE

~~A rise of 10% a year for 2 years is a rise of 20%.~~

Why it feels right: Two lots of 10% add up to 20%, and the sum looks like the rule for simple interest.

The test that breaks it: Work it out for 100 goats. After 2 years the herd is 121, not 120. The second rise is 10% of the bigger herd.

What is true: Multiply the multipliers: 1.1 × 1.1 = 1.21. That is a rise of 21%.

KEY WORDS

decay
a fall by the same percent every period. The multiplier is below 1.

growth
a rise by the same percent every period. The multiplier is above 1.

model
a rule that says how an amount changes, such as multiplying by 0.85 every year.

BACK TO YOUR GUESS

The answer is B: No. Each year it keeps 85% of the fish, so some always remain.

Each year the lake keeps 85% of the fish it had. Multiplying a number above zero by 0.85 never gives zero. In the model the fish get fewer and fewer, but never none. A real lake has whole fish, so the model is a guide while there are many.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which multiplier gives a fall of 8% each year?

   A 0.08

   B 1.08

   C 0.8

   D 0.92

   Answer at the back of the book.

2. A herd of 200 goats grows by 10% each year. How many goats are there after 2 years?

   A 242

   B 240

   C 220

   Answer at the back of the book.

3. An amount of 500 falls by 20% each year. What is it after 2 years?

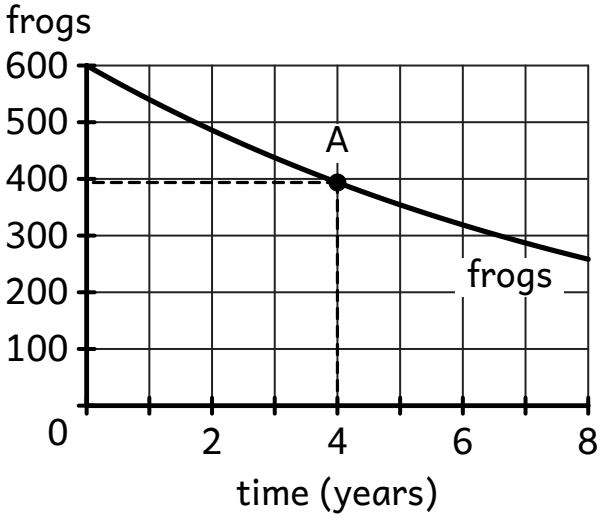
   A 300

   B 400

   C 320

   Answer at the back of the book.

4. The graph shows frogs in a pond. The number falls by 10% each year from 600. About how many frogs are there at point A, in year 4?



A 360

B 394

C 540

D 437

Answer at the back of the book.

5. Fish in a lake fall by 15% each year in the model. Will the number of fish ever reach exactly zero?

A No. It gets smaller each year but never reaches zero.

B Yes, after about 7 years.

C Yes, but only after 15 years.

Answer at the back of the book.

6. **From an earlier chapter**

Work out the simple interest on 2 500 coins at 4% a year for 3 years.

Answer at the back of the book.

7. A herd of 300 goats grows by 10% each year. How many goats are there after 2 years?

Answer at the back of the book.

8. The number of fish falls by 20% each year from 1 000. How many fish are there after 3 years?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A fall of p% a year multiplies by 1 − p/100 every year. A rise multiplies by 1 + p/100. Decay stays above zero. Growth bends upward.

Draw: A falling curve that flattens and a rising curve that steepens, each with its multiplier written beside it.

M4.2.5 CHAPTER 5

Harder ratio problems

> **Mei and Ana share beads in the ratio 2 : 3. Ana and Tariq share beads in the ratio 4 : 5. What is the ratio of all three?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Mei : Ana : Tariq is 2 : 3 : 5.
- ☐ **B** Mei : Ana : Tariq is 2 : 4 : 5.
- ☐ **C** Mei : Ana : Tariq is 8 : 12 : 15.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Ratio and proportion, M3.4.1. If simplifying a ratio such as 6 : 4 looks new, read this first.
- Level 3, Ratio and proportion, M3.4.3. If sharing 35 seeds in the ratio 2 : 5 looks new, read this first.
- Level 3, Negative numbers, factors and powers, M3.1.6. If finding a lowest common multiple looks new, read this first.

BY THE END YOU CAN

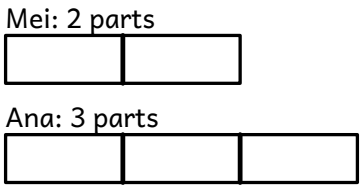
- Combine two ratios into one, like a : b : c
- Solve a ratio problem that changes when some are added

Mei and Ana share beads in the ratio 2 : 3. Ana and Tariq share beads in the ratio 4 : 5. Ana appears in both ratios. We want one ratio for all three.

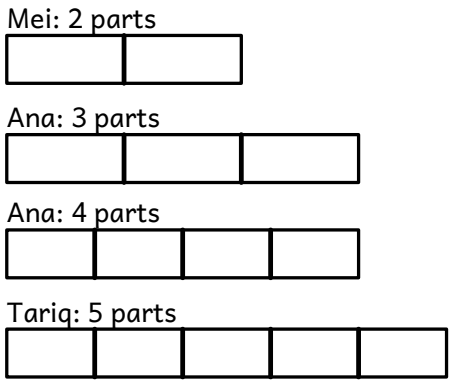
Ana is 3 parts in the first ratio and 4 parts in the second. Those are different sizes of part, so we cannot join the numbers yet. First we make Ana the same in both.

DRAW IT FIRST

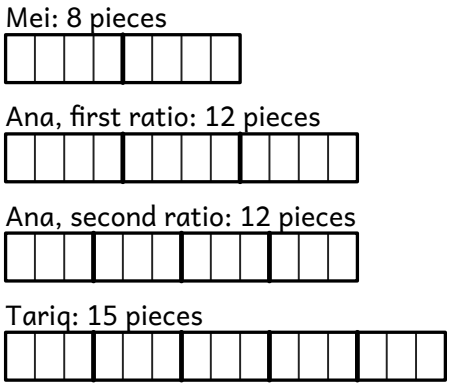
Make the middle the same



1 Mei and Ana: 2 parts to 3 parts. Ana has 3 parts here.



2 Ana and Tariq: 4 parts to 5 parts. Ana has 4 parts here, a different size of part.



3 Cut each part into smaller equal pieces. Ana's bar keeps its length. Make Ana 12 pieces in both. Mei is then 8 pieces and Tariq is 15 pieces.

Multiplying both numbers of a ratio by the same number gives an equal ratio. 2 : 3 = 8 : 12 when you multiply by 4. Pick the numbers so Ana matches: the **lowest common multiple** of 3 and 4, which is 12.

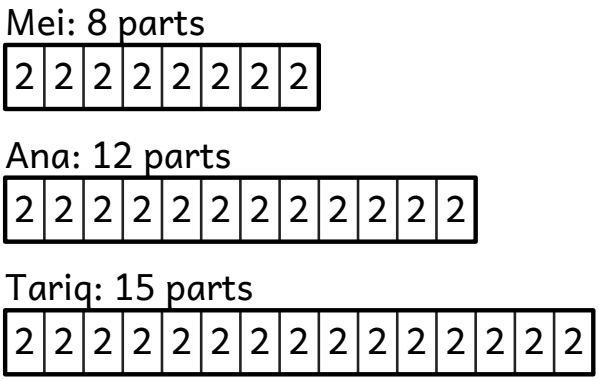
WORKED EXAMPLE

Mei : Ana is 2 : 3 and Ana : Tariq is 4 : 5. Find Mei : Ana : Tariq.

1. Ana has 3 parts in the first ratio and 4 parts in the second.
2. The lowest common multiple of 3 and 4 is 12. Make Ana 12 in both.
3. First ratio, multiply by 4: 2 : 3 = 8 : 12.
4. Second ratio, multiply by 3: 4 : 5 = 12 : 15.
5. Ana is 12 in both, so Mei : Ana : Tariq = 8 : 12 : 15.

Answer: 8 : 12 : 15

With *a* : b : c you can share an amount. Add the parts to find how many equal shares there are. Then find one part, and multiply.



70 beads in 35 parts. Each part is 2 beads.

WORKED EXAMPLE

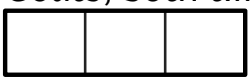
Mei, Ana and Tariq have 70 beads together in the ratio 8 : 12 : 15. How many beads does each have?

1. Add the parts: 8 + 12 + 15 = 35.
2. One part is 70 ÷ 35 = 2 beads.
3. Mei has 8 × 2 = 16, Ana has 12 × 2 = 24, and Tariq has 15 × 2 = 30.
4. Check: 16 + 24 + 30 = 70.

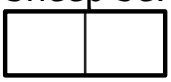
Answer: 16, 24 and 30 beads

Now a different kind of problem. Some are added, and the ratio changes. One group stays the same size. That group gives you the key.

Goats, both times: 3 parts



Sheep before: 2 parts



Sheep after: 5 parts



The goats stay 3 parts. The sheep go from 2 parts to 5 parts. The dotted parts are the 12 sheep that were added.

WORKED EXAMPLE

Kofi has goats and sheep in the ratio 3 : 2. He gets 12 more sheep, and now goats : sheep is 3 : 5. How many of each did he have at the start?

1. The goats did not change, so 3 parts of goats keep the same size.
2. The sheep went from 2 parts to 5 parts. That is 5 − 2 = 3 more parts.
3. The 3 more parts are 12 sheep, so one part is 12 ÷ 3 = 4.
4. At the start: goats 3 × 4 = 12, and sheep 2 × 4 = 8.
5. Check: after, 12 goats and 8 + 12 = 20 sheep, and 12 : 20 = 3 : 5.

Answer: 12 goats and 8 sheep

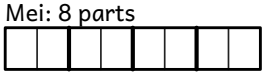
SEE IT THREE WAYS

With things

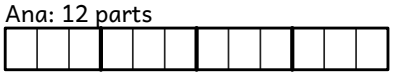
Mei has 2 beads for every 3 of Ana's. Put 4 such sets together. Mei has 8 beads and Ana has 12.

As a picture

Mei: 8 parts



Ana: 12 parts



In symbols

2 : 3 = 8 : 12.

TRY IT

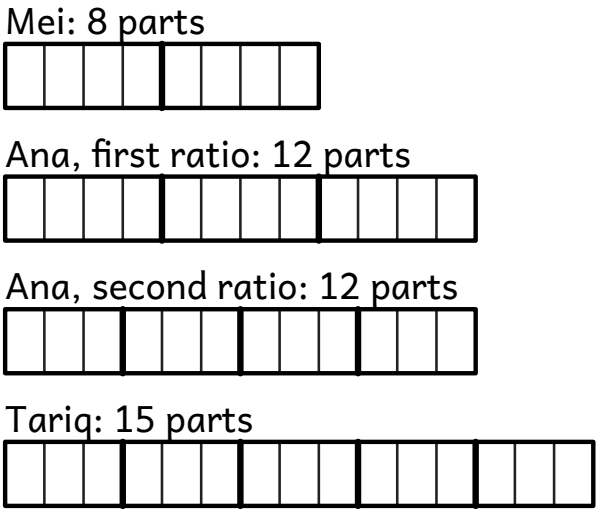
Check with stones

You need: 35 stones or beans.

1. Lay out 8 stones for Mei, 12 for Ana and 15 for Tariq, in three rows.
2. Group Mei's and Ana's stones in fours. Count the groups. Then group Ana's and Tariq's stones in threes, and count again.

What you will see: In fours, Mei has 2 groups and Ana has 3, so Mei : Ana is 2 : 3. In threes, Ana has 4 groups and Tariq has 5, so Ana : Tariq is 4 : 5.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~Mei : Ana is 2 : 3 and Ana : Tariq is 4 : 5, so Mei : Ana : Tariq is 2 : 3 : 5.~~

Why it feels right: The names Mei, Ana and Tariq come in order, and so it looks as if the numbers can come in order too.

The test that breaks it: In 2 : 3 : 5, Ana : Tariq would be 3 : 5. The question says Ana : Tariq is 4 : 5. That does not match.

What is true: The shared person must have the same number in both ratios. Make Ana 12 in both first, then join.

A WRONG IDEA MANY PEOPLE HAVE

~~Goats : sheep is 3 : 2. Adding 12 sheep makes it 3 : 14.~~

Why it feels right: The sheep number in the ratio is 2, and 12 sheep are added, so 2 + 12 looks right.

The test that breaks it: The 2 in the ratio is 2 parts, not 2 sheep. A part is not the same size as one sheep. You cannot add 12 sheep to 2 parts.

What is true: Find how big one part is first. Here one part is 4 sheep, so 12 sheep are 3 parts.

KEY WORDS

lowest common multiple
the smallest whole number above 0 that both numbers divide into. For 3 and 4 it is 12.

BACK TO YOUR GUESS

The answer is C: Mei : Ana : Tariq is 8 : 12 : 15.

Ana has 3 parts in one ratio and 4 parts in the other, so the numbers cannot just be joined. Make Ana the same in both: 12. Then 2 : 3 becomes 8 : 12, and 4 : 5 becomes 12 : 15. So Mei : Ana : Tariq is 8 : 12 : 15.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A : B is 3 : 4 and B : C is 2 : 5. What is A : B : C?

   A 3 : 4 : 5

   B 3 : 4 : 10

   C 6 : 8 : 5

   D 3 : 2 : 5

   Answer at the back of the book.

2. A : B is 2 : 5 and B : C is 10 : 3. What is A : B : C?

   A 2 : 5 : 3

   B 2 : 10 : 3

   C 4 : 5 : 3

   D 4 : 10 : 3

   Answer at the back of the book.

3. Share 90 beads in the ratio 2 : 3 : 4. How many beads is the largest share?

   A 40

   B 30

   C 10

   D 22.5

   Answer at the back of the book.

4. Red and blue beads are in the ratio 2 : 3. After 10 more blue beads are added, the ratio is 2 : 5. How many red beads are there?

   A 5

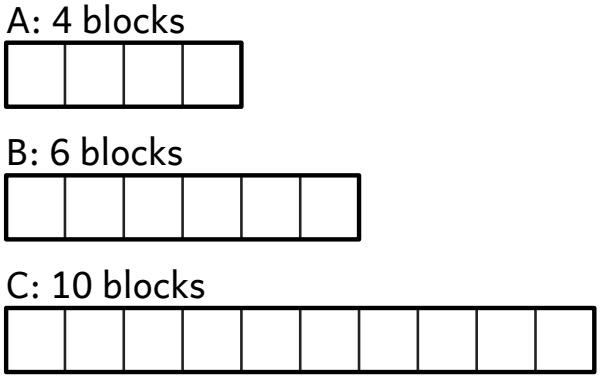
   B 15

   C 10

   D 25

   Answer at the back of the book.

5. The bars show A, B and C. What is the ratio A : B : C in its simplest form?



A 4 : 6 : 10

B 2 : 3 : 5

C 5 : 3 : 2

Answer at the back of the book.

6. **From an earlier chapter**

After a 20% rise an amount is 180. What was it before? Type the number only.

Answer at the back of the book.

7. Share 84 stones in the ratio 1 : 2 : 4. How many stones is the largest share?

Answer at the back of the book.

8. A : B is 3 : 5 and B : C is 10 : 7. Write A : B : C. Type the three numbers.

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

To join a : b and b : c, make b the same in both with the lowest common multiple. When some are added, find what stayed the same, then the size of one part.

Draw: Two pairs of bars with the shared bar cut into 12 equal pieces, and the joined bars 8 : 12 : 15 beneath.

M4.2.6 CHAPTER 6

Proportion as a formula

If y is proportional to x, how do I write that as a formula?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** y = kx, where k is a fixed number.
- ☐ **B** y = k + x, where k is a fixed number.
- ☐ **C** y = k ÷ x, where k is a fixed number.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Ratio and proportion, M3.4.5. If using a table of values to solve a proportion problem looks new, read this first.
- Level 3, Ratio and proportion, M3.4.8. If one amount going down as the other goes up looks new, read this first.
- Level 3, Letters for numbers and simple equations, M3.8.2. If finding the value of 3n + 2 when n = 4 looks new, read this first.

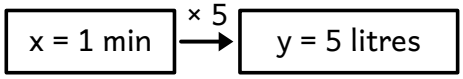
BY THE END YOU CAN

- Write y = kx for direct proportion and find k
- Solve problems where y is proportional to x^2 or to 1/x

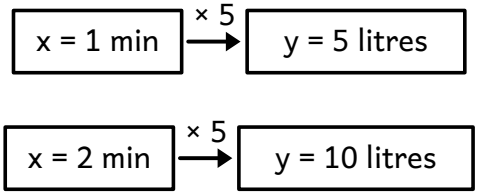
A tap fills 5 litres every minute. After 1 minute there are 5 litres. After 2 minutes there are 10. After 3 minutes there are 15. Each time, the litres are the minutes times 5.

DRAW IT FIRST

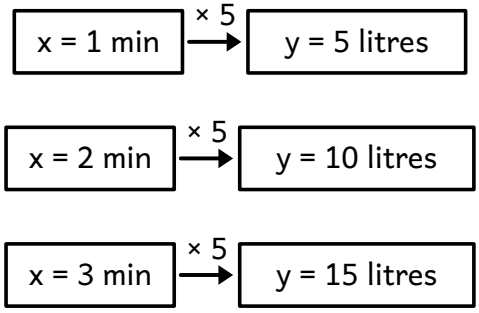
Every y is x times the same number



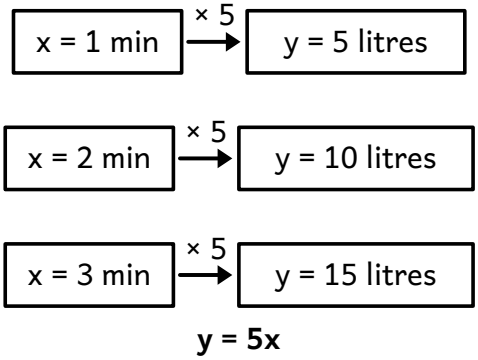
1 After 1 minute, y is 1 × 5 = 5.



2 After 2 minutes, y is 2 × 5 = 10.



3 After 3 minutes, y is 3 × 5 = 15.



4 The same number 5 multiplies every x. We write the rule as a formula: y = 5x.

We say y is **directly proportional** to x, or just proportional. It means y = kx, where k is a fixed number that never changes. The number k is the **constant of proportionality**. Here k = 5. It must not be 0.

To find k, take any pair of values and divide: k = y ÷ x. Then k stays the same for every pair.

x	y	y ÷ x
1	5	5
2	10	5
3	15	5

The last column is always 5. That is how you know y is proportional to x.

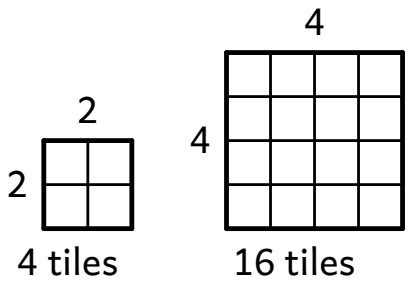
WORKED EXAMPLE

y is proportional to x. When x = 7, y = 35. Find the formula, and find y when x = 12.

1. Write the formula y = kx.
2. Put in x = 7 and y = 35: 35 = k × 7, so k = 35 ÷ 7 = 5.
3. The formula is y = 5x.
4. When x = 12, y = 5 × 12 = 60.

Answer: y = 5x. When x = 12, y = 60.

Some things grow faster than that. Take a right-angled triangle whose two shorter sides are both x cm. Its area is half of x times x. When x doubles, the area does not double. It becomes four times as big.

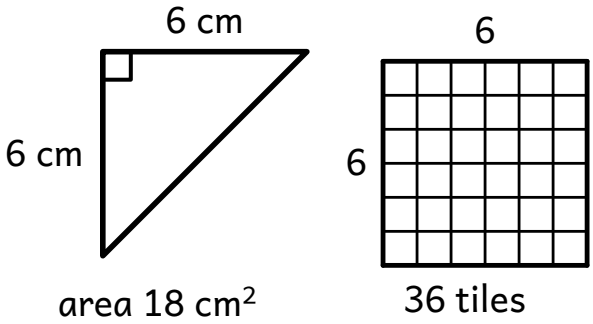


Double the side and the area is 2 × 2 = 4 times as big.

When y is proportional to x^2, the formula is $y = kx^2$. For the triangle, $y = 0.5x^2$. Doubling x multiplies y by $2^2 = 4$.

x (cm)	y (cm²)
2	2
4	8
6	18

The area y cm² of a right-angled triangle with two sides of x cm. Going from x = 2 to x = 4 multiplies y by 4. Going to x = 6 multiplies it by 9.



A right-angled triangle with two sides of 6 cm is half of a 6 cm by 6 cm square. The square holds 36 tiles of 1 cm², so the triangle has an area of 18 cm².

WORKED EXAMPLE

A right-angled triangle has two equal shorter sides of x cm. Its area y cm² is proportional to x^2. When x = 6, y = 18. Find y when x = 10.

1. Write the formula $y = kx^2$.
2. Put in x = 6 and y = 18: $18 = k \times 6^2$, so $k = 18 \div 36 = 0.5$.
3. The formula is $y = 0.5x^2$.
4. When x = 10, $y = 0.5 \times 10^2 = 0.5 \times 100 = 50$.

Answer: $y = 0.5x^2$. When x = 10, y = 50.

The other kind goes the opposite way. A 120 km journey takes 3 hours at 40 km per hour. At twice the speed it takes half the time. The time y hours is 120 divided by the speed x.

We say y is **inversely proportional** to x, or proportional to 1/x. The formula is y = k ÷ x. Multiply x and y and you always get k. Here x is a speed, so it is above 0.

km/h	Hours	Distance (km)
20	6	120
40	3	120
60	2	120
80	1.5	120

Speed is in km per hour. The last column is always 120 km. When the speed goes up, the time goes down.

WORKED EXAMPLE

The time y hours for a journey is inversely proportional to the speed x km per hour. At 40 km per hour it takes 3 hours. How long does it take at 60 km per hour?

1. Time is proportional to 1 ÷ speed, so y = k ÷ x.
2. At 40 km per hour the time is 3 hours: 3 = k ÷ 40.
3. Multiply both sides by 40: k = 3 × 40 = 120.
4. The formula is y = 120 ÷ x. Check: 120 ÷ 40 = 3.
5. At 60 km per hour: y = 120 ÷ 60 = 2.

Answer: 2 hours

SEE IT THREE WAYS

With things

A tap fills 5 litres each minute. After 1 minute: 5 litres. After 2 minutes: 10. After 3 minutes: 15.

As a picture

Minutes	Litres
1	5
2	10
3	15
4	20

In symbols

y = 5x.

TRY IT

Rectangles of the same area

You need: squared paper, a pencil.

1. Draw a rectangle 1 square across and 12 squares up. It holds 12 squares.
2. Draw more rectangles that also hold 12 squares: 2 across, 3 across, 4 across. Write how many squares go up each time.

What you will see: As the number across goes up, the number up goes down. Across times up is always 12. That is inverse proportion.

If you cannot try it, this is what happens.

Across	Up	Squares
1	12	12
2	6	12
3	4	12
4	3	12
6	2	12

A WRONG IDEA MANY PEOPLE HAVE

~~If y is proportional to x^2, then doubling x doubles y.~~

Why it feels right: Proportional usually means "when one doubles, the other doubles", and that is how direct proportion works.

The test that breaks it: Take $y = 0.5x^2$. When $x = 2$, $y = 2$. When $x = 4$, which is double, $y = 8$. That is four times as big, not double.

What is true: When y is proportional to x^2, doubling x multiplies y by $2^2 = 4$. Tripling x multiplies y by 9.

A WRONG IDEA MANY PEOPLE HAVE

~~Inverse proportion means y goes down by the same amount that x goes up.~~

Why it feels right: One goes up and the other goes down, so it feels as if they trade places evenly.

The test that breaks it: Take y = 24 ÷ x. When x = 4, y = 6. When x = 8, y = 3, which is a fall of 3. When x = 12, y = 2, which is a fall of only 1.

What is true: When x is multiplied by a number, y is divided by the same number. Then x times y stays the same.

KEY WORDS

directly proportional
y = kx for a fixed number k. Doubling x doubles y.

constant of proportionality
the fixed number k in y = kx or y = k ÷ x.

inversely proportional
y = k ÷ x for a fixed number k. Doubling x halves y.

BACK TO YOUR GUESS

The answer is A: y = kx, where k is a fixed number.

Proportional means y is always the same multiple of x. Double x and y doubles. So y = kx, with k a fixed number. The formula y = k ÷ x is for inverse proportion, where doubling x halves y.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. y is proportional to x. When x = 4, y = 28. Which formula is right?

   A $y = x + 24$

   B $y = 4x$

   C $y = 112x$

   D $y = 7x$

   Answer at the back of the book.

2. y is proportional to x. When x = 4, y = 10. Find y when x = 10.

   A 25

   B 16

   C 100

   Answer at the back of the book.

3. y is proportional to x^2. When x = 2, y = 12. Find y when x = 4.

   A 24

   B 192

   C 48

   Answer at the back of the book.

4. y is inversely proportional to x. When x = 4, y = 6. Find y when x = 8.

   A 12

   B 3

   C 2

   Answer at the back of the book.

5. A tank fills at 12 litres a minute and is full after 5 minutes. The time is inversely proportional to the speed. How long does it take at 15 litres a minute?

   A 4 minutes

   B 6.25 minutes

   C 2 minutes

   Answer at the back of the book.

6. y is proportional to x^2. When x = 3, y = 36. Which formula is right?

   A $y = 12x$

   B $y = 9x^2$

   C $y = 4x^2$

   Answer at the back of the book.

7. **From an earlier chapter**

   The number of goats falls by 20% each year from 400. How many goats are there after 2 years?

   Answer at the back of the book.

8. y is proportional to x. When x = 6, y = 18. Find y when x = 15.

   Answer at the back of the book.

9. y is proportional to x^2. When x = 3, y = 27. Find y when x = 5.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Proportional to x: y = kx, and y ÷ x is always k. Proportional to x^2: $y = kx^2$. Inversely proportional to x: y = k ÷ x, and x times y is always k.

Draw: Three small tables, one for each kind, with the column y ÷ x, y ÷ x^2 or x × y beside each.

M4.2.7 CHAPTER 7

Graphs of proportion

How can I tell from a graph that two amounts are in proportion?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** The graph is any straight line.
- ☐ **B** The graph is a straight line that goes through the origin.
- ☐ **C** The graph is a curve that goes down.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Coordinates and straight lines, M3.9.4. If plotting points and joining them with a line looks new, read this first.
- Level 3, Coordinates and straight lines, M3.9.8. If spotting direct proportion as a line through (0, 0) looks new, read this first.
- Level 3, Ratio and proportion, M3.4.8. If one amount going down as the other goes up looks new, read this first.

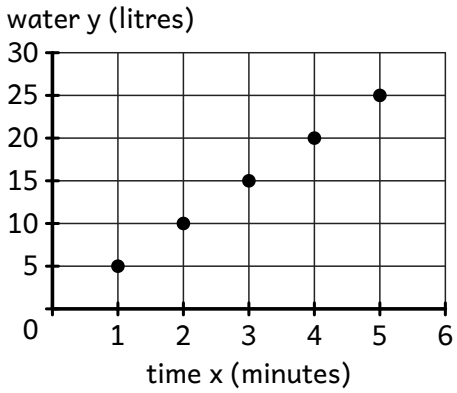
BY THE END YOU CAN

- Sketch the graphs of $y = kx$ and $y = k/x$
- Spot direct or inverse proportion from a table or graph

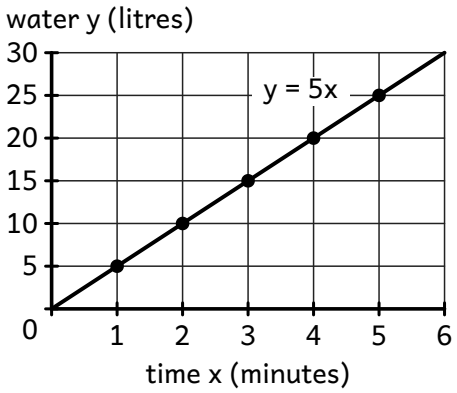
Ravi has a tap that fills 5 litres every minute. He plots minutes across and litres up. The rule is $y = 5x$.

DRAW IT FIRST

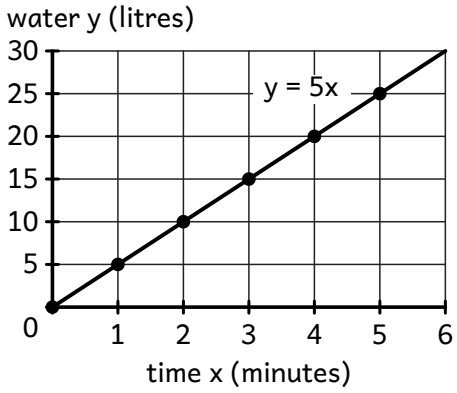
Plot the tap



1 Plot a dot for each minute: (1, 5), (2, 10), (3, 15), (4, 20), (5, 25).

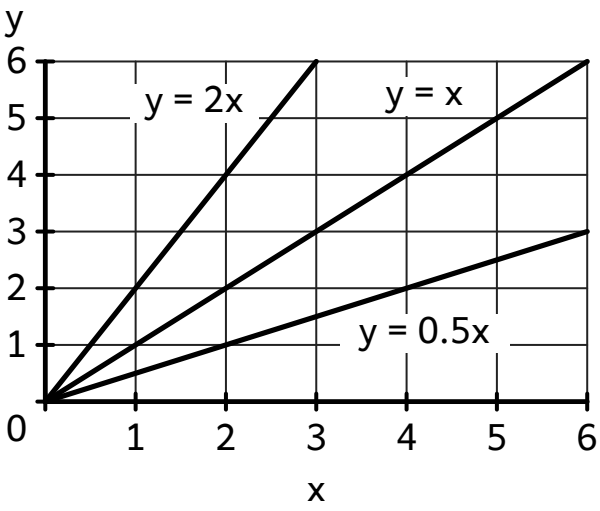


2 The dots lie in a straight line. Draw the line through them.



3 At 0 minutes there is 0 litres, so the line starts at (0, 0), the origin.

The graph of y = kx is always a straight line through the **origin**, the point (0, 0). The number k says how steep the line is. It is the **gradient**: how much y goes up for each 1 across. Here k = 5, so y goes up 5 for each 1 across.



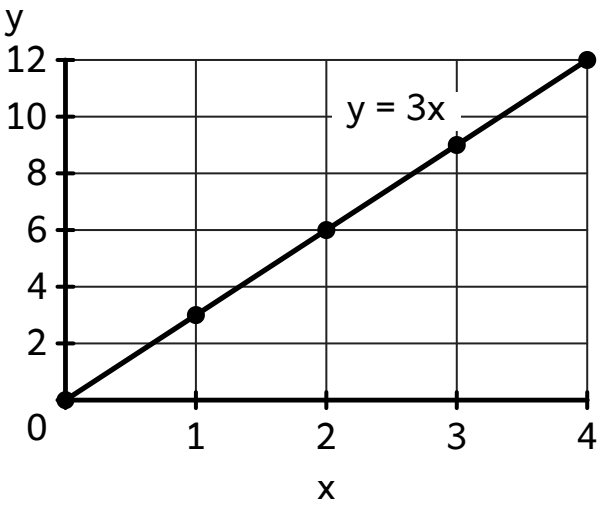
A bigger k gives a steeper line. All three lines pass through the origin.

WORKED EXAMPLE

Sketch the graph of y = 3x for x from 0 to 4.

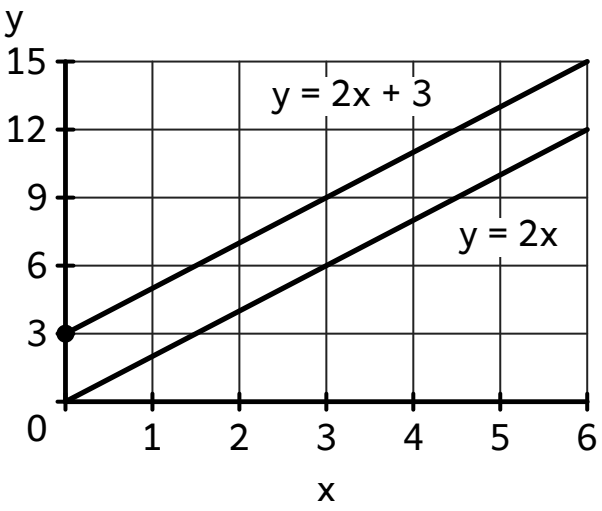
1. Choose some values of x: 0, 1, 2, 3 and 4.
2. Work out y = 3x for each: y = 0, 3, 6, 9 and 12. The last is y = 12.
3. Plot the points (0, 0), (1, 3), (2, 6), (3, 9) and (4, 12).
4. Join them with a straight line. It goes through the origin.

Answer: A straight line through (0, 0) and (4, 12)



The graph of y = 3x.

A straight line is not enough. Compare y = 2x with y = 2x + 3. The second line is straight, but it crosses the y axis at 3, not at 0. When x = 0 it gives y = 3. Doubling x does not double y, so it is not proportion.



Both lines are straight. Only the one through the origin is proportion.

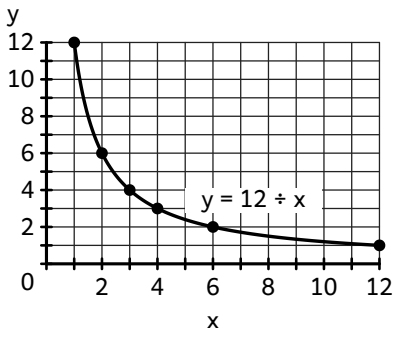
Now inverse proportion. Take y = 12 ÷ x, for x above 0. When x is 1, 2, 3, 4, y is 12, 6, 4, 3. The points do not lie in a line. They lie on a curve that falls.

SEE IT THREE WAYS

With things

Rectangles that hold 12 squares: 1 across and 12 up, 2 across and 6 up, 3 across and 4 up.

As a picture



In symbols

x × y = 12, so y = 12 ÷ x.

The curve gets very high near the y axis, and very close to the x axis far along. It comes as near as you like to each axis but never touches either. A y of 0 would need 12 ÷ x = 0, and no x gives that.

WORKED EXAMPLE

Sketch the graph of y = 12 ÷ x for x from 1 to 12.

1. Choose values of x that divide 12: 1, 2, 3, 4, 6 and 12.
2. When x = 2, y = 12 ÷ 2 = 6.
3. When x = 4, y = 12 ÷ 4 = 3.
4. When x = 6, y = 12 ÷ 6 = 2.
5. Plot the points and join them with a smooth curve. It falls and never touches the axes.

Answer: A smooth curve through (2, 6), (4, 3) and (6, 2)

You can also tell the two kinds apart from a table. For direct proportion, y ÷ x is the same every time. For inverse proportion, x × y is the same every time.

x	y	y ÷ x	x × y
2	3	1.5	6
4	6	1.5	24
6	9	1.5	54
8	12	1.5	96

Test a table: divide, then multiply. One column stays the same.

WORKED EXAMPLE

A table has x = 2, 4, 6, 8 and y = 3, 6, 9, 12. Is y proportional to x, inversely proportional, or neither?

1. Divide y by x for the first pair: 3 ÷ 2 = 1.5.
2. Divide for the others: 6 ÷ 4, 9 ÷ 6 and 12 ÷ 8 are all 1.5.
3. Now multiply: 2 × 3 = 6, but 4 × 6 = 24. This is not fixed.
4. y ÷ x is always 1.5, so y is proportional to x, with k = 1.5.

Answer: Directly proportional, with k = 1.5

TRY IT

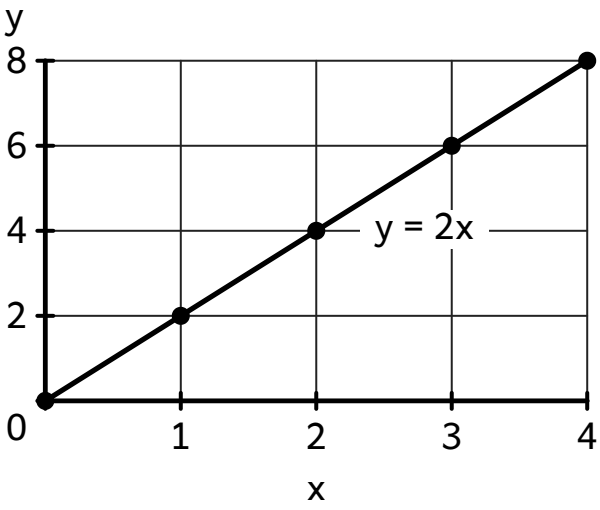
Plot y = 2x on squared paper

You need: squared paper, a ruler, a pencil.

1. Draw an x axis from 0 to 4 and a y axis from 0 to 8.
2. Plot the points (0, 0), (1, 2), (2, 4), (3, 6) and (4, 8).
3. Lay the ruler along the points and draw the line.

What you will see: The points lie on one straight line, and the line passes through the origin. Each step across goes up 2.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~Any straight line shows direct proportion.~~

Why it feels right: Direct proportion makes a straight line, so a straight line feels like proof.

The test that breaks it: Take y = 2x + 3. When x = 1, y = 5. When x = 2, which is double, y = 7. That is not double 5.

What is true: Direct proportion needs a straight line through the origin. Zero of x must give zero of y.

A WRONG IDEA MANY PEOPLE HAVE

~~If y goes down as x goes up, the graph shows inverse proportion.~~

Why it feels right: Inverse proportion does go down as x goes up, so any fall looks like it.

The test that breaks it: Take y = 10 − x. It falls as x goes up. When x = 2, y = 8, and x × y = 16. When x = 4, y = 6, and x × y = 24. The product is not fixed.

What is true: Inverse proportion needs x × y to stay the same. Its graph is a curve that never touches the axes, not a straight line.

KEY WORDS

origin
the point (0, 0), where the two axes cross.

gradient
how steep a line is: how much y goes up for each 1 across.

BACK TO YOUR GUESS

The answer is B: The graph is a straight line that goes through the origin.

Direct proportion makes a straight line through the origin, the point where both amounts are zero. A straight line that misses the origin is not proportion. A curve that falls and never touches the axes shows inverse proportion.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which description fits the graph of y = kx, with k above 0?

   A Any straight line

   B A curve that falls and never touches the axes

   C A straight line through the origin

   D A straight line that cuts the y axis at k

   Answer at the back of the book.

2. The table shows x and y. Which is it?

x	y
2	8
3	12
5	20
6	24

   A Direct proportion, y = 4x

   B Inverse proportion

   C Neither

   Answer at the back of the book.

3. The table shows x and y. Which is it?

x	y
1	16
2	8
4	4
8	2

   A Direct proportion

   B Inverse proportion, y = 16 ÷ x

   C Neither

   Answer at the back of the book.

4. The table shows x and y. Which is it?

x	y
1	4
2	6
3	8
4	10

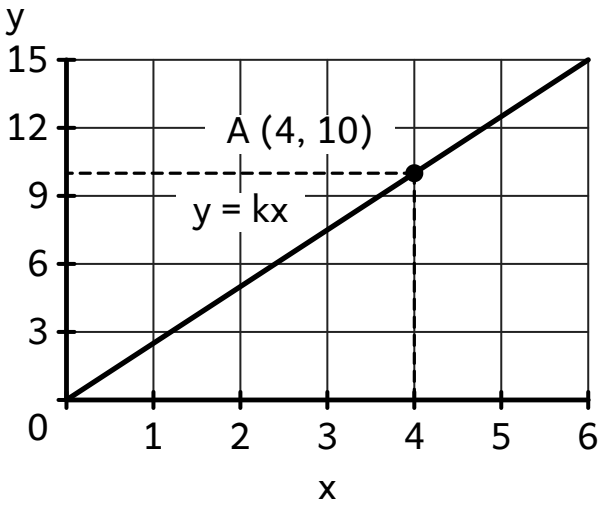
A Direct proportion

B Inverse proportion

C Neither

Answer at the back of the book.

5. y is proportional to x. The graph is a straight line through the origin and through the point A (4, 10). What is k?



A 0.4

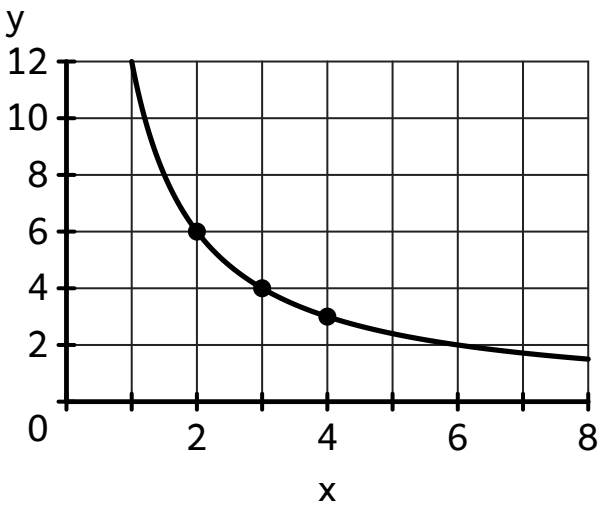
B 6

C 10

D 2.5

Answer at the back of the book.

6. The curve shows inverse proportion. It passes through (2, 6), (3, 4) and (4, 3). Which formula is right?



A $y = 12 \div x$

B $y = 3x$

C $y = 8 - x$

Answer at the back of the book.

7. **From an earlier chapter**

y is inversely proportional to x. When $x = 5$, $y = 6$. Find y when $x = 3$.

Answer at the back of the book.

8. y is proportional to x. The graph is a straight line through the origin and through the point (6, 21). What is k?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

The graph of y = kx is a straight line through the origin. The graph of y = k ÷ x is a curve that never touches the axes. A table: y ÷ x fixed is direct, x × y fixed is inverse.

Draw: A straight line through the origin beside a falling curve, with y ÷ x and x × y written under them.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

compound interest
interest worked out on the amount so far, including interest already added. M4.2.3

constant of proportionality
the fixed number k in y = kx or y = k ÷ x. M4.2.6

decay
a fall by the same percent every period. The multiplier is below 1. M4.2.4

directly proportional
y = kx for a fixed number k. Doubling x doubles y. M4.2.6

gradient
how steep a line is: how much y goes up for each 1 across. M4.2.7

growth
a rise by the same percent every period. The multiplier is above 1. M4.2.4

interest
extra amount added to savings each year, worked out as a percent. M4.2.3

inversely proportional
y = k ÷ x for a fixed number k. Doubling x halves y. M4.2.6

lowest common multiple
the smallest whole number above 0 that both numbers divide into. For 3 and 4 it is 12. M4.2.5

model
a rule that says how an amount changes, such as multiplying by 0.85 every year. M4.2.4

multiplier
the number you multiply by to change an amount, such as 1.12 for a rise of 12%. M4.2.1

origin
the point (0, 0), where the two axes cross. M4.2.7

original amount
the amount before the change. It is 100%. M4.2.2

percent
out of 100. So 12% is 12 out of 100, or 0.12. M4.2.1

percentage change
the change divided by the original amount, times 100. M4.2.1

simple interest
interest that is always a percent of the start amount, so it is the same every year. M4.2.3

Answers

M4.2.1. Percentage increase and decrease

1. **1.08**

   A rise of 8% makes 100% + 8% = 108% of the start, which is 1.08 as a decimal.

   - **If you chose A:** This is 8% of the amount, not the amount plus 8%. A rise keeps the whole 100% and adds 8% to it, so 1 + 0.08. Go to M4.2.1.
   - **If you chose B:** Read the decimal carefully. 1.8 is 1 + 0.8, which is a rise of 80%. A rise of 8% is 1 + 0.08. Go to M4.2.1.
   - **If you chose D:** This is the multiplier for a fall of 8%. A rise adds to 1. Go to M4.2.1.

2. **450**

   360 × 1.25 = 450. The 25% rise is 90, and 360 + 90 = 450.

   - **If you chose B:** You added 25 to 360. A rise of 25% is a quarter of 360 added on, which is 90. Go to M4.2.1.
   - **If you chose C:** This is 25% of 360, the size of the rise. The question asks for the new amount, which is 360 plus 90. Go to M4.2.1.
   - **If you chose D:** This is 360 decreased by 25%. The arrow says × 1.25, which is bigger than 1, so the amount grows. Go to M4.2.1.

3. **130**

   A fall of 35% leaves 65%, so the multiplier is 0.65. 200 × 0.65 = 130.

   - **If you chose A:** You took 35 away from 200. A fall of 35% takes away 35 out of every 100, which is 70 from 200. Go to M4.2.1.
   - **If you chose B:** This is 35% of 200, the size of the fall. The question asks for what is left: 200 minus 70. Go to M4.2.1.
   - **If you chose C:** This adds 35% instead of taking it away. A fall uses a multiplier below 1. Go to M4.2.1.

4. **12%**

   The change is 56 − 50 = 6. Then 6 ÷ 50 × 100 = 12.

   - **If you chose A:** The change is 6, but 6 is not yet a percent. Divide 6 by the original, 50, and multiply by 100. Go to M4.2.1.
   - **If you chose C:** You divided the change by the new amount, 56. A percentage change is measured against the original, 50. Go to M4.2.1.
   - **If you chose D:** The new amount is 112% of the original, but that is the whole amount. The change is only the 12% above the 100%. Go to M4.2.1.

5. **A fall of 25%**

   The change is 160 − 120 = 40. Then 40 ÷ 160 × 100 = 25. The amount fell.

   - **If you chose A:** You divided the change, 40, by the new amount, 120. Divide by the original amount, 160. Go to M4.2.1.
   - **If you chose B:** 120 is 75% of 160, so 75% is what is left. The fall is the other 25%. Go to M4.2.1.
   - **If you chose C:** The new amount is smaller than the original, so the amount fell. A rise needs a bigger new amount. Go to M4.2.1.

6. **184**

   A rise of 15% makes 115% of the start, so the multiplier is 1.15. 160 × 1.15 = 184.

7. **15**

   The change is 46 − 40 = 6 cm. Then 6 ÷ 40 × 100 = 15. The plant grew by 15%.

M4.2.2. Finding the original amount

1. **150**

   A rise of 40% is a multiplier of 1.4. Divide to go back: 210 ÷ 1.4 = 150.

   - **If you chose A:** You took 40% off 210. The 40% rise was worked out on the original amount, which is smaller than 210, so divide by 1.4. Go to M4.2.2.
   - **If you chose B:** You took 40 away from 210. The rise is 40% of the original, not the number 40. Go to M4.2.2.
   - **If you chose C:** You multiplied by 1.4 again. That makes the amount bigger, but the original was smaller than 210. Go to M4.2.2.

2. **80**

   A fall of 20% leaves 80%, a multiplier of 0.8. Divide to go back: 64 ÷ 0.8 = 80.

   - **If you chose A:** You added 20% of 64. The fall was 20% of the original, which is more than 64, so divide by 0.8. Go to M4.2.2.
   - **If you chose C:** You added 20 to 64. The fall is 20% of the original, not the number 20. Go to M4.2.2.
   - **If you chose D:** You multiplied by 0.8 again, which is a second fall. The original was bigger than 64. Go to M4.2.2.

3. **120**

   A fall of 25% leaves 75%, a multiplier of 0.75. 90 ÷ 0.75 = 120.

   - **If you chose A:** You added 25% of 90. The fall was 25% of the original, which is more than 90, so divide by 0.75. Go to M4.2.2.
   - **If you chose B:** You added 25 to 90. The fall is 25% of the original, not the number 25. Go to M4.2.2.
   - **If you chose D:** You took 25% off 90, which is a second fall. The original was bigger than 90. Go to M4.2.2.

4. **0.96**

   Multiply the two multipliers: 1.2 × 0.8 = 0.96.

   - **If you chose B:** A rise and a fall of the same percent do not cancel. The fall is a percent of a bigger amount than the rise. Go to M4.2.2.
   - **If you chose C:** This is the multiplier of the fall alone. The rise happened first, so multiply by 1.2 as well. Go to M4.2.2.

5. **4% less than the start**

   The end is 1.2 × 0.8 = 0.96 of the start. That is 4% less.

   - **If you chose A:** They do not cancel. The fall of 20% is taken from the bigger, risen amount, so it takes off more than the rise added. Go to M4.2.2.
   - **If you chose B:** The end multiplier is 0.96, which is below 1. An amount that has been multiplied by less than 1 is smaller. Go to M4.2.2.
   - **If you chose C:** The rise comes first and makes up for most of the fall. The end is much nearer the start than 20% less. Go to M4.2.2.

6. **25**

   The change is 75 − 60 = 15. Then 15 ÷ 60 × 100 = 25.

7. **300**

   A rise of 15% is a multiplier of 1.15. Divide to go back: 345 ÷ 1.15 = 300.

M4.2.3. Simple and compound interest

1. **600**

   One year: 6% of 2 500 = 150. For 4 years: 4 × 150 = 600.

   - **If you chose A:** This is one year of interest. Simple interest is the same every year, so multiply by the number of years, 4. Go to M4.2.3.
   - **If you chose B:** This is the compound interest, rounded. The question says simple, which is the same amount every year. Go to M4.2.3.
   - **If you chose D:** This is the total, the start plus the interest. The question asks for the interest only. Go to M4.2.3.

2. **4 410**

   Multiply by 1.05 twice: 4 000 × 1.05 = 4 200, and 4 200 × 1.05 = 4 410.

   - **If you chose B:** This is simple interest: 5% of 4 000 is 200 a year, and two years give 400. Compound interest works the second year on 4 200, not 4 000. Go to M4.2.3.
   - **If you chose C:** This is the amount after one year only. The question asks for two years, so multiply by 1.05 a second time. Go to M4.2.3.

3. **1.04**

   A rise of 4% each year is a multiplier of 1 + 0.04 = 1.04.

   - **If you chose A:** This is 4% of the amount, which is the interest alone. The multiplier keeps the whole amount and adds the interest: 1 + 0.04. Go to M4.2.1.
   - **If you chose C:** This is 1 + 0.4, a rise of 40%. A rise of 4% is 1 + 0.04. Go to M4.2.1.

4. **Plan B**

   Plan B bends upward, so it gains more each year than the year before. That is interest on interest, which is compound. Plan A is straight, so it adds the same each year.

   - **If you chose A:** Plan A is a straight line: it adds the same amount each year, which is simple interest. Go to M4.2.3.
   - **If you chose B:** Only one graph bends. A graph that bends upward gains more each year than the year before, and that is compound. Go to M4.2.3.
   - **If you chose C:** You can. A straight line adds the same each year. A curve that rises faster and faster means interest on interest. Go to M4.2.3.

5. **They have the same**

   In year 1 both clubs work out the interest on the start amount, so they add the same. From year 2 the compound club earns interest on that interest as well.

   - **If you chose A:** In year 1 there is no interest yet to earn interest, so compound has nothing extra. The difference starts in year 2. Go to M4.2.3.
   - **If you chose B:** Simple is never ahead. In year 1 the two are equal, and after that compound is ahead. Go to M4.2.3.

6. **600**

   A fall of 20% is a multiplier of 0.8. Divide to go back: 480 ÷ 0.8 = 600.

7. **360**

   One year: 3% of 3 000 = 90. For 4 years: 4 × 90 = 360.

8. **2 420**

   Multiply by 1.1 twice: 2 000 × 1.1 = 2 200, and 2 200 × 1.1 = 2 420.

M4.2.4. Growth and decay

1. **0.92**

   A fall of 8% leaves 100% − 8% = 92%, which is 0.92 as a decimal.

   - **If you chose A:** This is 8% of the amount, which is the size of the fall. The amount that is left is 100% − 8% = 92%. Go to M4.2.1.
   - **If you chose B:** This is the multiplier for a rise of 8%. A fall uses a multiplier below 1. Go to M4.2.1.
   - **If you chose C:** This is 80%, a fall of 20%. Read the decimal carefully: 8% leaves 92%, which is 0.92. Go to M4.2.1.

2. **242**

   200 × 1.1 = 220 after one year, and 220 × 1.1 = 242 after two.

   - **If you chose B:** You added 10% of 200 twice. The second rise is 10% of the new herd, 220, not of 200. Go to M4.2.4.
   - **If you chose C:** This is the herd after one year. The question asks for two years, so multiply by 1.1 again. Go to M4.2.4.

3. **320**

   500 × 0.8 = 400 after one year, and 400 × 0.8 = 320 after two.

   - **If you chose A:** You took 20% of 500 off twice. The second fall is 20% of what is left after year 1, which is only 400. Go to M4.2.4.
   - **If you chose B:** This is the amount after one year. The question asks for two years, so multiply by 0.8 again. Go to M4.2.4.

4. **394**

   The multiplier is 0.9. Four years: $600 \times 0.9^4 = 393.66$, which is about 394 frogs.

   - **If you chose A:** You took 10% of 600 off four times, which is 60 each time. Each fall is 10% of what is left, so the falls get smaller. Go to M4.2.4.
   - **If you chose C:** This is the number after one year. Point A is in year 4, after four falls. Go to M4.2.4.
   - **If you chose D:** This is the number after three years. Point A is in year 4, so multiply by 0.9 once more. Go to M4.2.4.

5. **No. It gets smaller each year but never reaches zero.**

   Each year the number is multiplied by 0.85. A number above zero multiplied by 0.85 is still above zero. So the model never reaches zero.

   - **If you chose B:** Seven falls of 15% do not add up to 100%. Each fall is 15% of what is left, which is smaller each year. After 7 years about a third of the fish remain. Go to M4.2.4.
   - **If you chose C:** There is no year when the multiplier 0.85 makes the number zero. A number above zero times 0.85 is still above zero. Go to M4.2.4.

6. **300**

   One year: 4% of 2 500 = 100. For 3 years: 3 × 100 = 300.

7. **363**

   300 × 1.1 = 330 after one year, and 330 × 1.1 = 363 after two.

8. **512**

   The multiplier is 0.8. 1 000 × 0.8 = 800, then 800 × 0.8 = 640, then 640 × 0.8 = 512.

M4.2.5. Harder ratio problems

1. **3 : 4 : 10**

   B is 4 and 2. The lowest common multiple is 4. The first ratio stays 3 : 4. The second, multiplied by 2, is 4 : 10. So A : B : C is 3 : 4 : 10.

   - **If you chose A:** You joined the numbers without matching B. B is 4 in the first ratio and 2 in the second, so they are not the same size of part. Go to M4.2.5.
   - **If you chose C:** You doubled the first ratio but not the second. B is 8 in the first, and 2 in the second, which is not 8. Scale both ratios. Go to M4.2.5.
   - **If you chose D:** You took A from the first ratio and B : C from the second. B is 4 in one and 2 in the other, so the parts are different sizes. Make B the same first. Go to M4.2.5.

2. **4 : 10 : 3**

B is 5 and 10. The lowest common multiple is 10. The first ratio, multiplied by 2, is 4 : 10. The second ratio stays 10 : 3. So A : B : C is 4 : 10 : 3.

- **If you chose A:** You joined the numbers without matching B. B is 5 in the first ratio and 10 in the second. Go to M4.2.5.
- **If you chose B:** You took B from the second ratio but did not scale A with it. If B doubles from 5 to 10, A must double too. Go to M4.2.5.
- **If you chose C:** You doubled A but kept B at 5. Doubling the first ratio makes 4 : 10, so B must be 10. Go to M4.2.5.

3. **40**

The parts add up to 2 + 3 + 4 = 9. One part is 90 ÷ 9 = 10. The largest share is 4 × 10 = 40.

- **If you chose B:** This is the middle share, 3 parts. The largest share is the 4 parts. Go to M3.4.3.
- **If you chose C:** This is the size of one part, 90 ÷ 9. The largest share is 4 of those parts. Go to M3.4.3.
- **If you chose D:** You divided 90 by 4, the largest number in the ratio. Divide by the total of the parts, 2 + 3 + 4 = 9, then take 4 of them. Go to M3.4.3.

4. **10**

The red beads did not change. Blue went from 3 parts to 5 parts, which is 2 more parts. So 2 parts are 10 beads, and one part is 5. Red is 2 parts, so 2 × 5 = 10.

- **If you chose A:** This is the size of one part. There are 2 parts of red, so red is 2 × 5. Go to M4.2.5.
- **If you chose B:** This is the number of blue beads at the start, 3 parts. The question asks for red. Go to M4.2.5.
- **If you chose D:** This is the number of blue beads at the end, 5 parts. The question asks for red. Go to M4.2.5.

5. **2 : 3 : 5**

The bars hold 4, 6 and 10 blocks. All three divide by 2, giving 2, 3 and 5. No bigger number divides all three, so 2 : 3 : 5 is simplest.

- **If you chose A:** This is a true ratio for the bars, but not the simplest. Divide each number by 2 to get the simplest form. Go to M3.4.1.
- **If you chose C:** This is the right numbers in the wrong order. A is the first bar, so its number comes first. Go to M3.4.1.

6. **150**

   A rise of 20% is a multiplier of 1.2. Divide to go back: 180 ÷ 1.2 = 150.

7. **48**

   The parts add up to 1 + 2 + 4 = 7. One part is 84 ÷ 7 = 12. The largest share is 4 × 12 = 48.

8. **6 : 10 : 7**

   B is 5 and 10. The lowest common multiple is 10. The first ratio, multiplied by 2, is 6 : 10. The second stays 10 : 7. So A : B : C is 6 : 10 : 7.

M4.2.6. Proportion as a formula

1. **y = 7x**

   k = y ÷ x = 28 ÷ 4 = 7. So y = 7x.

   - **If you chose A:** This fits x = 4, y = 28, but adding is not proportion. When x doubles to 8 this gives 32, and y should double to 56. Go to M4.2.6.
   - **If you chose B:** You used the value of x as k. The number k is y ÷ x, which is 28 ÷ 4. Go to M4.2.6.
   - **If you chose C:** You multiplied 4 by 28. To find k you divide y by x. Go to M4.2.6.

2. **25**

   k = 10 ÷ 4 = 2.5. Then y = 2.5 × 10 = 25.

   - **If you chose B:** You added 6 to y because x went up by 6. Proportion multiplies: y is always the same number times x. Go to M4.2.6.
   - **If you chose C:** You multiplied 10 by 10. First find k = y ÷ x = 10 ÷ 4, then multiply k by the new x. Go to M4.2.6.

3. **48**

   k = 12 ÷ 2^2 = 3. Then y = 3 × 4^2 = 3 × 16 = 48.

   - **If you chose A:** You doubled y because x doubled. That is direct proportion with x. Here y is proportional to x^2, so doubling x makes y four times as big. Go to M4.2.6.
   - **If you chose B:** You multiplied 12 by 4^2. But y = 12 already includes the 2^2. Find k first: k = 12 ÷ 2^2, then use k × 4^2. Go to M4.2.6.

4. **3**

   k = x × y = 4 × 6 = 24. Then y = 24 ÷ 8 = 3.

   - **If you chose A:** You doubled y because x doubled. In inverse proportion, doubling x halves y. Go to M4.2.6.
   - **If you chose C:** You took 4 away from 6 because x went up by 4. Inverse proportion divides: x times y stays the same. Go to M4.2.6.

5. **4 minutes**

   The tank holds 12 × 5 = 60 litres. At 15 litres a minute it takes 60 ÷ 15 = 4 minutes.

   - **If you chose B:** You multiplied the time by 15 ÷ 12, as if it were direct proportion. A faster tap needs less time, not more. Go to M4.2.6.
   - **If you chose C:** You took 3 away from 5 because the tap is 3 litres faster. Inverse proportion divides: k = 12 × 5 = 60 litres, so the time is 60 ÷ 15. Go to M4.2.6.

6. **$y = 4x^2$**

   k = y ÷ x^2 = 36 ÷ 3^2 = 36 ÷ 9 = 4. So $y = 4x^2$.

   - **If you chose A:** You divided y by x, not by x^2. That gives k for y = kx, which is direct proportion with x. Here y is proportional to x^2. Go to M4.2.6.
   - **If you chose B:** You used x^2 itself as k. The number k is y ÷ x^2, which is 36 ÷ 9. Go to M4.2.6.

7. **256**

   The multiplier is 0.8. 400 × 0.8 = 320 after one year, and 320 × 0.8 = 256 after two.

8. **45**

   k = 18 ÷ 6 = 3. Then y = 3 × 15 = 45.

9. **75**

   k = 27 ÷ 3^2 = 27 ÷ 9 = 3. Then y = 3 × 5^2 = 3 × 25 = 75.

M4.2.7. Graphs of proportion

1. **A straight line through the origin**

   When x = 0, y = k × 0 = 0, so the line passes through the origin. All the points are in a straight line because y goes up by k for each 1 across.

   - **If you chose A:** A straight line that misses the origin, such as y = 2x + 3, is not proportion. When x is 0, y must be 0. Go to M4.2.7.
   - **If you chose B:** That is the graph of y = k ÷ x, inverse proportion. The graph of y = kx rises in a straight line. Go to M4.2.7.
   - **If you chose D:** k is the steepness of the line, not where it crosses the y axis. The line y = kx crosses the y axis at 0. Go to M4.2.7.

2. **Direct proportion, y = 4x**

   y ÷ x is 8 ÷ 2 = 4 for every pair, so y = 4x.

   - **If you chose B:** In inverse proportion y goes down when x goes up. Here y goes up with x, so it is not inverse. Go to M4.2.6.
   - **If you chose C:** Divide y by x for each pair: 8 ÷ 2, 12 ÷ 3, 20 ÷ 5 and 24 ÷ 6 are all 4. A fixed answer means direct proportion. Go to M4.2.6.

3. **Inverse proportion, y = 16 ÷ x**

   x × y is 1 × 16 = 16 for every pair, so y = 16 ÷ x.

   - **If you chose A:** In direct proportion y goes up when x goes up. Here y goes down as x goes up. Go to M4.2.6.
   - **If you chose C:** Multiply x by y for each pair: 1 × 16, 2 × 8, 4 × 4 and 8 × 2 are all 16. A fixed answer means inverse proportion. Go to M4.2.6.

4. **Neither**

   Dividing y by x gives 4, 3, 2.67 and 2.5, which is not fixed. The product x × y is 4, 12, 24 and 40, which is not fixed either. The pattern is y = 2x + 2, a straight line that misses the origin.

   - **If you chose A:** Divide y by x: 4 ÷ 1 = 4, but 6 ÷ 2 = 3. The answer is not fixed. Also, when x = 0 this pattern would give y = 2, not 0. Go to M4.2.6.
   - **If you chose B:** In inverse proportion y goes down as x goes up. Here y goes up with x, and x × y is 4, 12, 24 and 40. Go to M4.2.6.

5. **2.5**

   k = y ÷ x = 10 ÷ 4 = 2.5. For every 1 across, the line goes up 2.5.

   - **If you chose A:** You divided x by y. The number k is y ÷ x, which is 10 ÷ 4. Go to M4.2.6.
   - **If you chose B:** You took 4 away from 10. Proportion divides: k = y ÷ x. Go to M4.2.6.
   - **If you chose C:** 10 is the y value of A. k is how much y goes up for each 1 across, which is 10 ÷ 4. Go to M4.2.6.

6. **y = 12 ÷ x**

   x × y is 2 × 6 = 12, 3 × 4 = 12, 4 × 3 = 12. The product is always 12, so y = 12 ÷ x.

   - **If you chose B:** This fits (2, 6), but when x = 3 it gives 9, not 4. It is also a rising line, not a falling curve. Go to M4.2.6.
   - **If you chose C:** This fits (2, 6), but when x = 3 it gives 5, not 4. A straight line is not inverse proportion. Check x × y for each point. Go to M4.2.6.

7. **10**

   k = x × y = 5 × 6 = 30. Then y = 30 ÷ 3 = 10.

8. **3.5**

   k = y ÷ x = 21 ÷ 6 = 3.5.

About this book

For learners who can find a percentage of an amount, share in a ratio and use a letter for a number. In many schools this fits teenagers of about 14 to 16, and adults who want to refresh percentages, ratio and proportion can start here too. Each chapter takes one or two sessions. A learner can work alone: every exercise has its answer and the reason for it.

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